Conclusion

A new ferrite absorption modulator has been developed for high-speed switching of microwave power. This high-speed amplitude modulator makes use of a longitudinally magnetized ferrite rod (split along its length) centrally located inside a standard rectangular waveguide excited in its fundamental TE_{01} mode. A thin resistive film, placed between the sections of the split ferrite rod and perpendicular to the input RF electric field, is used to attenuate the perpendicular mode generated in the magnetized ferrite rod.

Several fiber-glass waveguide models of this absorption modulator, with a 0.0005-inch thick silver plating on the inside, have been designed for sine-wave modulation frequencies up to 100 kc. These units make use of a modulating solenoid (with ferrite-rod loading) which is self-resonant at the desired modulating frequency. Because of the small electrical inertia associated with its low-field solenoid, these ferrite modulators can be designed for switching microwave energy in less than 1 μsec with noncritical magnetic control fields.

Other important applications of the absorption modulator described above include an electrically controlled variable attenuator for automatically stabilizing the amplitude of FM oscillators, pulse shaping of the microwave energy and high-speed TR switching in microwave radar systems.

Rectangular Waveguide Theoretical CW Average Power Rating*

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Summary—**A theoretical CW average power rating, limitation imposed by a temperature rise resulting from power dissipation within the rectangular waveguide walls, can be determined by predicting the rise in temperature. Formulas for the evaluation of the CW average power rating have been developed and are presented here, and the power rating curves are given for the WR-2300 waveguide (320 Mc) through the WR-19 waveguide (60 kMc).**

Localized hot spots, associated with a standing wave on a mismatched waveguide, require a derating factor. The axial flow of heat from these high current spots has been considered in calculating and plotting this derating factor.

I. Introduction

THE major power rating consideration in present waveguide designs, using the TE_{10} mode, has been the limitation imposed by voltage breakdown (peak power rating). However, with the advent of high average power microwave systems, the average power rating of waveguides should be considered for future designs. The average power rating is determined by imposing a specified temperature rise in the conducting wall of the waveguide. High average RF power can be obtained by systems using high power generators or an array of generators united through a power combiner.

This paper presents formulas and curves that can be used to determine the average power rating of rectangular waveguides.[1] The rating is defined by choosing an arbitrary limit for the temperature rise with the waveguide in an environment of still air. (The average power rating can be raised, if desired, by choosing a higher temperature limit and using forced cooling.)

High current points, associated with a voltage-standing wave, on a mismatched waveguide cause additional increases in temperature. Therefore, a derating factor as a function of the standing wave ratio (VSWR) has been plotted. This derating factor takes into account the axial flow of heat from the high current point.

II. Theory

A. Power Rating of a Waveguide

The approach to determining the average power rating is to find the attenuation per unit length. With the attenuation known, the dissipated power in the walls of the matched waveguide may be found where a given power is being transmitted through it. The dissipated

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[1] The average power rating of coaxial cable has been treated by W. W. Macalpine, "Heating of radio frequency cables," *Elec. Commun.*, vol. 25, pp. 84–89; March, 1948.

power is then related to the rate of heat transfer from the waveguide walls to obtain the power rating of the waveguide.

The formula for the attenuation[2] of an air-filled copper rectangular waveguide (TE_{10} mode) at a temperature of 20°C is

$$\alpha = \frac{12.68(10^{-5}) \frac{\lambda}{\lambda_c}\left[\frac{a}{2b} + \left(\frac{\lambda}{\lambda_c}\right)^2\right]}{\lambda^{3/2}\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}} \text{ db/ft}, \tag{1}$$

where

a = the wide inner dimension of the waveguide in meters,

b = the narrow inner dimension of the waveguide in meters,

λ = the wavelength in meters, and

$\lambda_c = 2a$.

The attenuation, in terms of the power levels in the waveguide, is

$$\alpha = 4.34 \frac{P_l}{P_1} \text{ db/unit length}, \tag{2}$$

where

P_1 = power input, or the power rating of the waveguide, and

P_l = power loss in the unit length of waveguide.

P_l is equivalent to the rate of heat transfer, q (Btu/hr[3]) from a unit length of the waveguide, and is related by $P_l = 0.293\ q$. Thus, the average power rating of the waveguide is

$$P_1 = \frac{1.271q}{\alpha}. \tag{3}$$

Neglecting transfer of heat by conduction, the rate of heat transfer is the sum of heat transferred by thermal convection and by thermal radiation, or

$$q = q_c + q_r \text{ Btu/hr}. \tag{4}$$

In the Appendix, Section B, the convection term is found to be

$$q_c = (\Delta T)^{5/4}\left[\frac{0.708A_b}{b'^{1/4}} + \frac{0.717A_a}{a'^{1/4}}\right] \text{Btu/hr/ft} \tag{5}$$

and the radiation term is found to be

$$q_r = 5.19(10^{-10})A_t(T_w^4 - T_a^4) \text{ Btu/hr/ft} \tag{6}$$

where

ΔT = temperature differential $= T_w - T_a$ in °F,

T_w = wall temperature of the waveguide in °R,

T_a = ambient temperature of the waveguide in °R,

a' = outer wide dimension of the waveguide in ft,

b' = outer narrow dimension of the waveguide in ft, and

$A_t = 2(A_a + A_b)$ = total outer surface area of 1 ft of waveguide in sq ft.

Eqs. (5) and (6) use the empirical heat transfer coefficients, and are theoretically valid for a rectangular waveguide. Also, the wave guide b dimension is in the horizontal position as illustrated in Fig. 1; the waveguide is in still air at sea level; and the thermal radiation factor or emissivity of the guide wall is equal to 0.3 (a black surface has an emissivity = 1). An additional assumption is that the dissipation of power per unit area in the waveguide walls is uniform for all walls.[4]

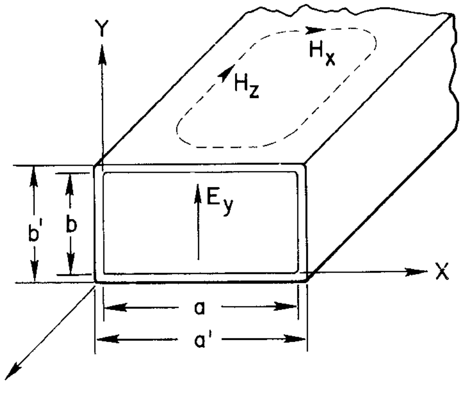

Fig. 1—Position of waveguide for determining heat transfer relations.

Eq. (4) gives the heat transfer by convection and by radiation, computed separately. In many practical cases, the treatment of convection and radiation as a single combined process is desirable. Using only a first-power equation, the heat transmission is

$$q = h_{rc}A_t(T_w - T_a) \text{ Btu/hr}, \tag{7}$$

where h_{rc} = the combined radiation and convection conductance, and can be found theoretically by equating (4) and (7), or h_{rc} can be found empirically.

B. Derating Factor for a VSWR on Waveguide

The power rating as determined from (4) is based on a matched waveguide. However, in many cases a VSWR exists on the waveguide. In the Appendix, the equation for temperature rise resulting from a reflecting termination is developed as a function of distance along the waveguide. Because high-current points exist on a mismatched waveguide, the heat at the hot spots will be conducted axially by virtue of the temperature dif-

[2] G. L. Ragan, "Microwave Transmission Circuits," M.I.T. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 9, p. 55; 1948.

[3] 1 Btu/hr = 0.293 watt.

[4] See Appendix, Section A.

ference. If the power rating of the waveguide is defined by a maximum permissible temperature rise, the temperature at the high-current points will be the limiting factor.

Two cases should be considered when discussing the derating factor for a VSWR on the waveguide. Case 1 represents the derating factor for a condition where the same amount of power is delivered to either the mismatched or to the matched loads. Case 2 is the derating factor for a condition where the incident wave is equal regardless of the load impedance. Case 1 is useful when a transmitter is delivering equal power to either a matched antenna or a mismatched antenna, and Case 2 is used in a balanced duplexer circuit or a power equalizer circuit[5] where the reflected wave does not disturb the loading of the transmitter. For example, in a duplexer circuit the net power delivered to the mismatched load (TR tube) approaches zero when the duplexer is in the transmit condition.

Case 1—Equal Power Delivered to Either Matched or Mismatched Loads: The general equation in terms of VSWR (see Appendix) for the waveguide temperature rise at the high-current point on the waveguide is

$$\Delta T_{\rho 1} = q\left[\frac{1}{h_{rc}A_t}\,\frac{\rho^2+1}{2\rho} + \frac{1}{kA_c\left(\frac{4\pi}{\lambda}\right) + h_{rc}A_t}\,\frac{\rho^2-1}{2\rho}\right], \quad (8)$$

where

q = heat transmission with a matched line,
h_{rc} = the combined radiation and convection conductance in Btu/(hr)(sq ft) °F,
A_t = total outer surface area of one foot length of waveguide, in square feet
A_c = cross-sectional area of waveguide in square feet,
k = heat conductivity of waveguide wall in (Btu)(ft)/(hr)(sq ft) °F,
ρ = VSWR, and
λ = wavelength in feet.

The term $kA_c(4\pi/\lambda)$ represents the heat transfer by conduction in the axial direction. Should the wall of the waveguide be composed of several laminated materials, such as copper-clad steel or metallic plating on fibreglas, the composite heat conductivity can be treated in the same manner as electrical conductivity; that is,

$$kA_c = k_1A_{c1} + k_2A_{c2} + \cdots,$$

where the subscript numbers refer to the different materials of the waveguide wall.

[5] R. W. Masters, "A power-equalizing network for antennas," Proc. IRE, vol. 37, pp. 735–738; July, 1949.

If $k=0$, then (8) reduces to

$$\Delta T_{\rho 1} = \frac{q\rho}{h_{rc}A_c}\,. \quad (9)$$

Eq. (9) indicates that, if no axial heat flow is involved and the same amount of power is delivered for both the mismatched load and the matched load cases, the temperature rise would be proportional to the VSWR. In most of the practical applications, the wall of the guide is copper, aluminum, or brass. Thus,

$$kA_c\left(\frac{4\pi}{\lambda}\right) \gg h_{rc}A_t,$$

and (8) reduces to

$$\Delta T_{\rho 1} \approx q\left[\frac{1}{h_{rc}A_t}\,\frac{\rho^2+1}{2\rho} + \frac{1}{kA_c\left(\frac{4\pi}{\lambda}\right)}\,\frac{\rho^2-1}{2\rho}\right]. \quad (10)$$

Eq. (10) represents the temperature rise (Case 1) at the high-current point of a waveguide with any VSWR on the line, if the waveguide wall is constructed of a relatively high thermal conductive material.

Case 2—Equal Incident Wave for Either Matched or Mismatched Loads: The general equation in terms of VSWR (see Appendix) for the waveguide temperature rise at the high-current point on the waveguide is

$$\Delta T_{\rho 2} = q\left[\frac{2}{h_{rc}A_c}\,\frac{(\rho^2+1)}{(\rho+1)^2} + \frac{2}{kA_c\left(\frac{4\pi}{\lambda}\right) + h_{rc}A_c}\left(\frac{\rho-1}{\rho+1}\right)\right], \quad (11)$$

where again q equals heat transmission with a matched line and the other symbols are defined under (8). The axial heat conductivity term kA_c has the same significance in Case 2 as in Case 1. If $k=0$, then (11) reduces to

$$\Delta T_{\rho 2} = \frac{q}{h_{rc}A_c}\,\frac{4\rho^2}{(\rho+1)^2}\,. \quad (12)$$

Eq. (12) indicates, that for a very high VSWR, the derating factor is asymptotic to 4. This value of 4 is reasonable, considering that, for a given incident voltage on a matched waveguide, the maximum voltage on the waveguide for a reflection coefficient of unity will be doubled. However, in Case 2 (unlike Case 1) the power delivered to the mismatched load is less than the power delivered to the matched load. As the VSWR increases, the power delivered to the load approaches zero.

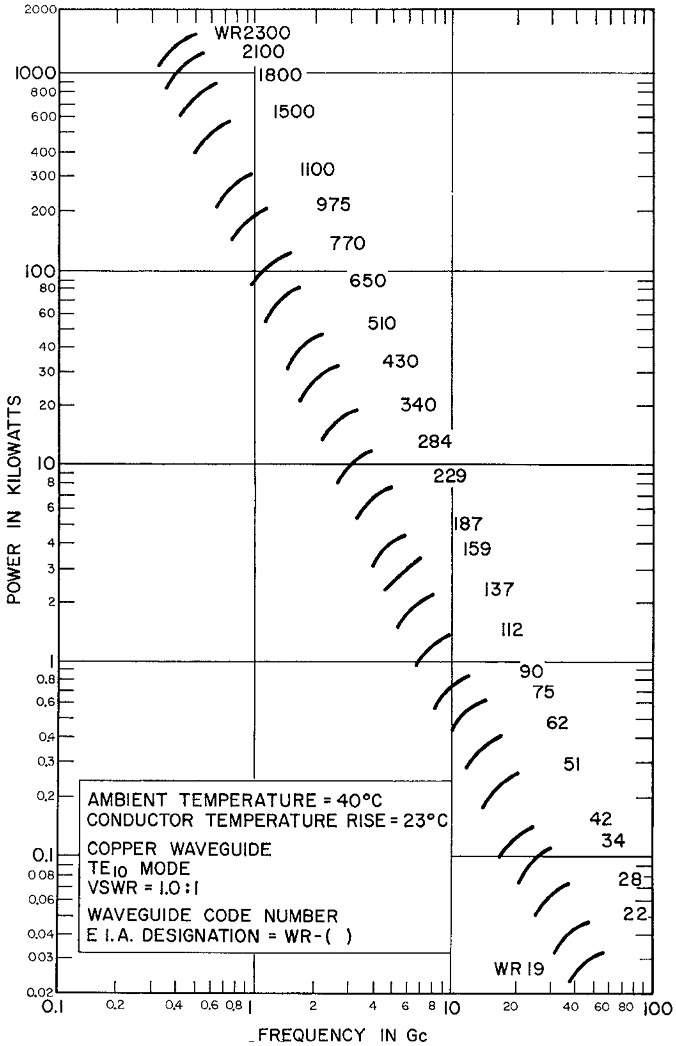

Fig. 2—Theoretical curves of the average power rating for copper rectangular waveguide (conductor temperature rise of 23°C).

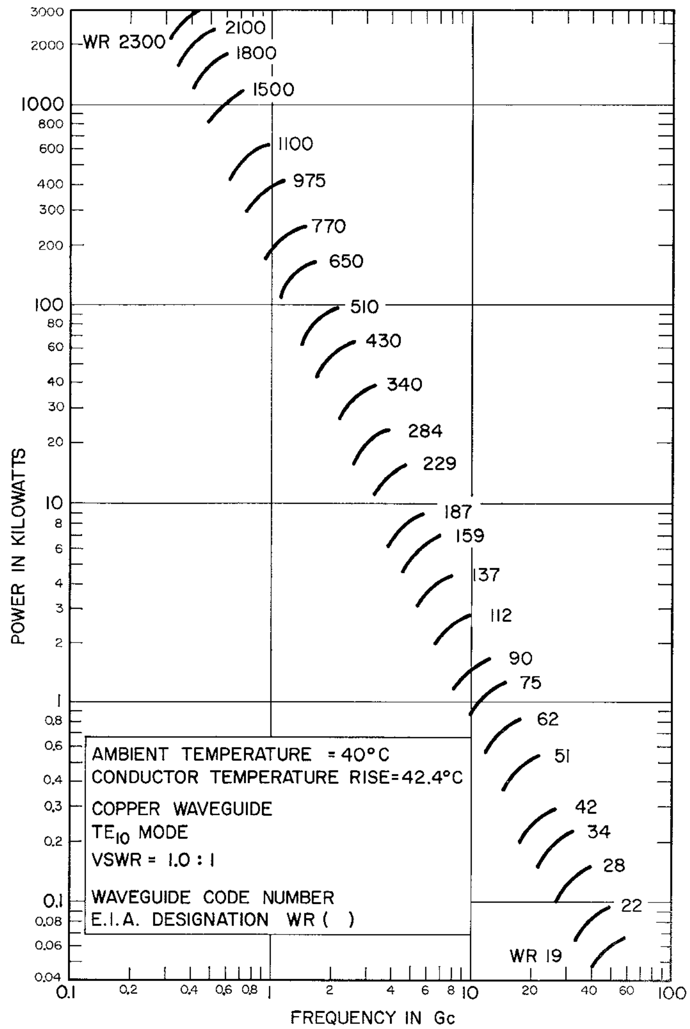

Fig. 3—Theoretical curves of the average power rating for copper rectangular waveguide (conductor temperature rise of 42.4°C).

Considering most of the practical cases where the wall of the guide is copper, aluminum, or brass, (11) reduces to

$$\Delta T_{\rho 2} \approx 2q\left[\frac{1}{h_{rc}A_c}\,\frac{(\rho^2+1)}{(\rho+1)^2}+\frac{1}{kA_c\left(\dfrac{4\pi}{\lambda}\right)}\left(\frac{\rho-1}{\rho+1}\right)\right]. \quad (13)$$

The derating factor (DF), because of the VSWR on the waveguide, is then

$$\mathrm{DF} = \frac{\Delta T}{\Delta T_{\rho 1}} \quad \text{or} \quad \frac{\Delta T}{\Delta T_{\rho 2}}, \quad (14)$$

where ΔT = temperature rise of the matched waveguide.

III. Description of Curves

A. Power Rating of Standard Rectangular Waveguides

Upon calculating the attenuation with (1), the heat transfer by convection and radiation is calculated for various temperature rises using (4). Then α and q are substituted into (3), giving the power ratings of the waveguide. Figs. 2–5 are curves for chosen temperature rises of 23°C, 42.4°C, 62°C, and 110°C, respectively, and represent the theoretical average power ratings of standard rectangular copper waveguide in an ambient temperature of 40°C. Eq. (1), the attenuation, has to be modified for the elevated waveguide wall temperatures of 63°C, 82.4°C, 102°C, and 150°C to determine the final power rating.

The theoretical peak power rating of standard waveguides (limitation due to voltage breakdown) with a dry air and unpressured environment is also plotted in Fig. 5. A comparison of the two curves in this figure will show that the CW power handling limitation is a function of the temperature rise, rather than the limitation imposed by voltage breakdown.

Figs. 2–5 can be modified for other ambient temperatures by referring to the ambient correction factor curves of Fig. 6. The correction factor is approximately the same for small or large size waveguides.

Experimental confirmation of the theoretical average power rating has been reported by Gould.[6] In recent

[6] L. Gould, Microwave Associates, Inc., private communication; September 19, 1960.

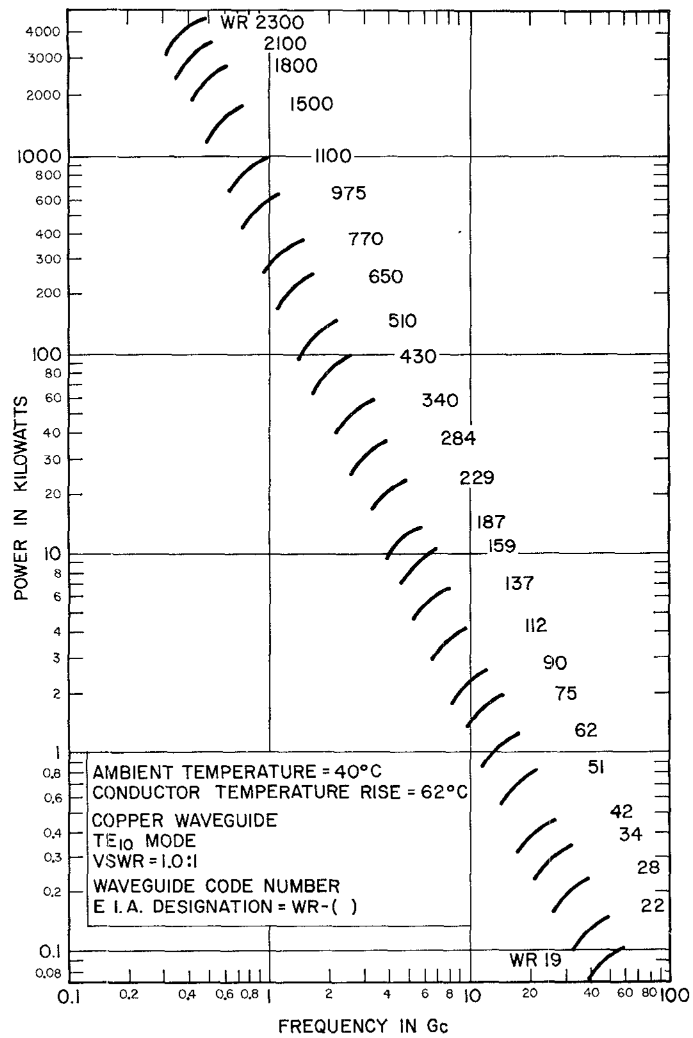

Fig. 4—Theoretical curves of the average power rating for copper rectangular waveguide (conductor temperature rise of 62°C).

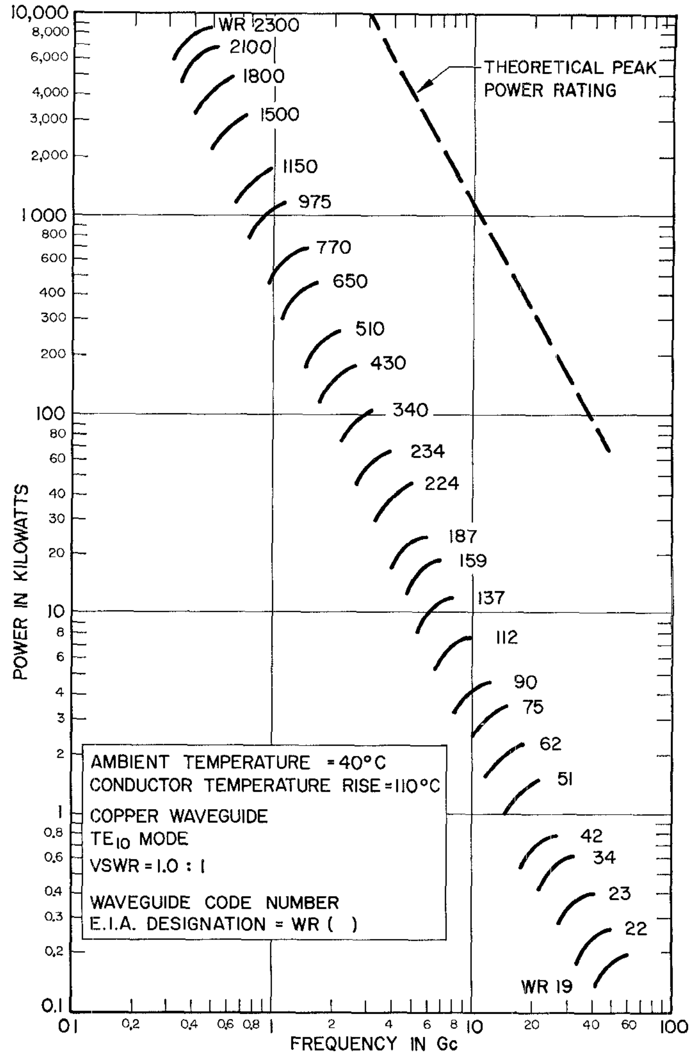

Fig. 5—Theoretical curves of the average power rating for copper rectangular waveguide (conductor temperature rise of 110°C).

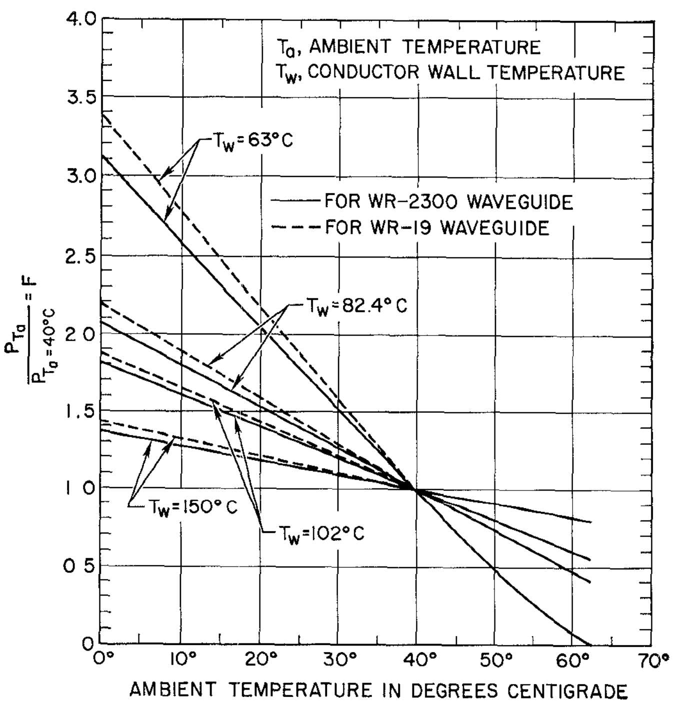

Fig. 6—Correction factor curve for average power rating of rectangular waveguide for various ambient temperatures and conductor wall temperatures.

high-power tests at 30 kw average power at 3000 Mc, Gould reports that the test yielded a waveguide conductor temperature rise of 60°C above room temperature, as compared to a theoretical value of 62.4°C. The comparison can be made by studying Fig. 3—the power rating curve for an ambient temperature of 40°C and a temperature rise of 42.4°C—and the ambient temperature correction curve of Fig. 6.

B. Combined Radiation and Convection Conductance

Often, it is desirable and convenient to use the combined radiation and convection conduction term. The h_{rc} is determined by using (7) with q equal to the heat calculated from (4). Fig. 7 is a plot of h_{rc} for various temperature differentials and waveguide sizes.

C. Derating Factor Caused by VSWR

Case 1—Equal Power Delivered to Load Regardless of Load Impedance: Fig. 8 plots the derating factor for a copper waveguide as a function of VSWR. The thermal conductivity term, k, for copper is 220 [Btu (ft)/(hr) (sq ft) °F]. Fig. 8 also shows the curve of the derating factor for $k=0$, the condition when the derating factor is proportional to the VSWR.

Case 2—Equal Incident Wave Regardless of Load Impedance: Fig. 9 is a plot of a copper waveguide derating factor as a function of the VSWR, when the incident wave remains the same amount for all values of VSWR. Fig. 9 also shows the derating factor curve for $k=0$, where ΔT_{p2} is calculated from (12).

For all practical purposes, in Cases 1 and 2, brass and aluminum waveguides yield the same results as the copper waveguide. Eqs. (8) and (13) should be used whenever the wall of the waveguide is made of a low conductivity material.

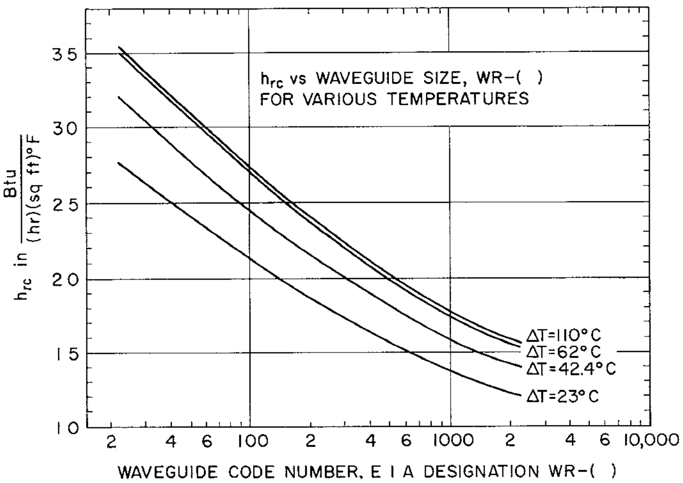

Fig. 7—Combined thermal radiation and convection conductance term as a function of waveguide size and temperature.

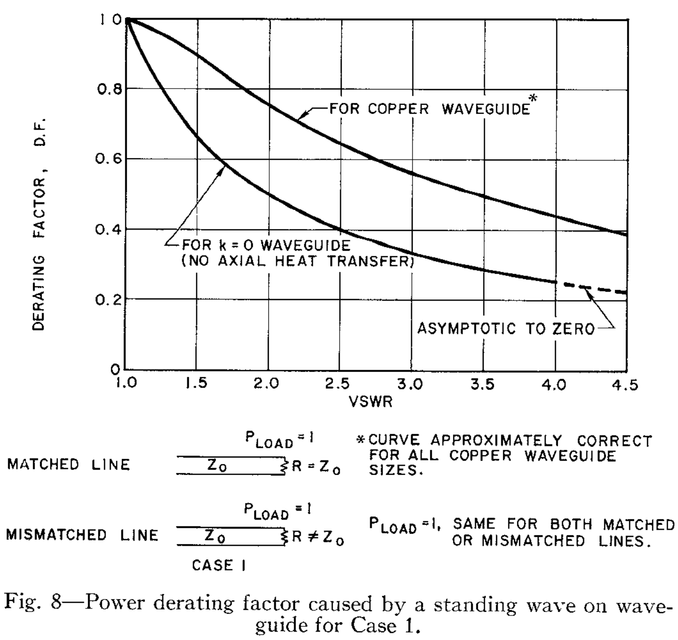

Fig. 8—Power derating factor caused by a standing wave on waveguide for Case 1.

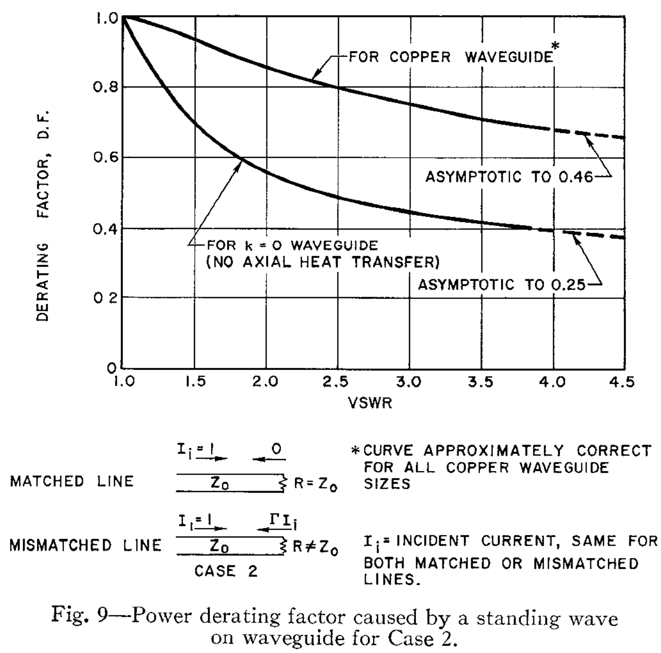

Fig. 9—Power derating factor caused by a standing wave on waveguide for Case 2.

IV. Conclusions

The CW average power rating of a rectangular waveguide is dictated by the permissible temperature rise above ambient of the waveguide wall. The theoretical power rating has been determined by calculating the convection, radiation, and conduction rate of heat transfer for both a mismatched load and a matched load on the waveguide. When a given power is delivered to a waveguide with a specified VSWR, the waveguide wall temperature rise along the axial direction can be predicted.

With the data presented in this paper, waveguide guide designers can determine, in advance, whether forced cooling will be necessary and can establish CW average power ratings for future designs.

Appendix

A. *Losses in Waveguide*

It is interesting to compare the relative power loss in the side walls with the loss in the top and bottom walls.

The average power absorbed by the walls of a waveguide may be found by integrating over a unit area the component of the Poynting vector that is directed into the waveguide walls. The current flow in the guide walls of low-loss waveguides may be determined in the same manner as in the ideal waveguide; *i.e.*, the current is equal, by the $\bar{n} \times \bar{H}$ rule, to the tangential magnetic fields at the boundary.

The power lost per unit length for the side walls of an air-filled waveguide[7] is

$$W_b = \frac{1}{2} R_s \int_0^b |H_z|^2 dy = \frac{R_s E^2}{2} \frac{\pi^2 b}{\eta^2 \beta^2 a^2}, \tag{15}$$

and for the top and bottom walls is

$$W_a = \frac{1}{2} R_s \int_0^a (|H_x|^2 + |H_z|^2) dx = \frac{R_s E^2}{2} \frac{a}{2\eta^2}, \tag{16}$$

where

R_s = surface resistivity of the conducting walls,

$$|H_z| = \frac{\pi E}{\eta \beta a} \cos \frac{\pi x}{a},$$

$$|H_x| = \frac{\sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}}{\eta} E \sin \frac{\pi x}{a},$$

$$\beta = \frac{2\pi}{\lambda}, \text{ and}$$

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}}.$$

[7] For waveguide field expressions, see, for example, S. Ramo and J. R. Whinnery, "Fields and Waves in Modern Radio," John Wiley and Sons, Inc., New York, N. Y., p. 370; 1953.

The total power loss per unit length of waveguide is

$$P_l = W_a + W_b = \frac{R_s E^2}{2\eta^2} \left[1 + \frac{2b}{a}\left(\frac{\lambda^2}{2a}\right)\right]. \tag{17}$$

The first term in the bracket is associated with losses on the top and bottom walls, and the last term is associated with losses in the side walls.

For conventional rectangular waveguide ($b/a \approx 1/2$), the ratio of power loss in the side walls to the loss in the top and bottom walls at approximately the mid-band frequency of the TE_{10} mode is

$$\frac{W_b}{W_a} \approx \frac{1}{2}.$$

Thus, the power dissipated in the waveguide per unit area is approximately the same in the top and bottom walls as in the side walls.

B. *Heat Transfer Convection and Radiation Term*[8]

The position of the rectangular waveguide for determining the heat transfer is shown in Fig. 1. The rates of heat transfer for both the convection and radiation terms from the four walls of the waveguide are treated separately, and the resulting temperature rises are added together. The convection and radiation flowrate expressions may be approximated by a linear temperature difference so the superposition theorem can be applied.

The thermal convection rate equation,

$$\frac{q}{A} = h_c(T_w - T_a) \frac{\text{Btu}}{\text{hr (sq ft)}}, \tag{18}$$

states that the thermal convection per unit transfer area q/A is proportional to the temperature difference $(T_w - T_a)$, the temperature of the waveguide wall less that of the external ambient temperature. The proportionality factor is known as the unit thermal convective conductance in

$$\frac{\text{Btu}}{\text{hr (sq ft) °F}}.$$

The thermal convection of the four walls of the waveguide is added to yield the total waveguide convection heat transfer, or

$$q_c = [2\,{}_N h_c A_b + {}_T h_c A_a + {}_B h_c A_a](T_w - T_a), \tag{19}$$

where

A_a = the top or bottom area of 1-ft length of waveguide (outer dimensions) in sq ft, and

A_b = the side or narrow area of 1-ft length of waveguide (outer dimensions) in sq ft.

[8] "Heating Ventilating Air Conditioning Guide," Am. Soc. Heating, Refrigerating and Air-Conditioning Engrs.; 1960.

The unit convective conductance for the four walls of the waveguide can be found in the literature.[9] For the side walls, or the narrow dimension of the guide.

$${}_N h_c = 0.354\left(\frac{T_w - T_a}{b'}\right)^{1/4};$$

for the top wall,

$${}_T h_c = 0.478\left(\frac{T_w - T_a}{a'}\right)^{1/4};$$

and for the bottom wall,

$${}_B h_c = 0.239\left(\frac{T_w - T_a}{a'}\right)^{1/4}.$$

The dimensions a' and b' are, respectively, the wide and narrow dimensions in feet of the waveguide outer walls. With the conductances substituted into (19), the rate of convection heat transfer for a rectangular waveguide is expressed by (5).

C. Derating Factor Caused by VSWR

With a reflecting termination on the waveguide, the standing wave pattern (Fig. 10) is the sum of the incident and reflected waves. The resultant current, neglecting losses, is

$$|I|^2 = |I_i|^2(1 + |\Gamma|^2 - 2|\Gamma| \cos(\psi - 2\beta z)), \quad (20)$$

where

$|I_i|$ = incident current,
Γ = reflection coefficient = $|\Gamma| < \psi$, and
z = distance along the waveguide.

The power loss P_l in a unit length of waveguide is proportional to I^2R; thus $P_l = q$ Btu/hr/unit length of waveguide. Therefore the standing wave pattern can be expressed in terms of the heat transfer, or

$$\begin{aligned} q &= q_i(1 + |\Gamma|^2 - 2|\Gamma| \cos(\psi - 2\beta z)) \\ &= q' - q'' \cos(\psi - 2\beta z), \end{aligned} \quad (21)$$

where q_i is defined as the "incident" heat, Btu/hr/unit length,

$$q' = q_i(1 + |\Gamma|^2) \text{ and}$$
$$q'' = 2q_i|\Gamma|.$$

The factor q' is independent of z, and no heat flows in the axial direction. The temperature rise above ambient temperature caused by q' is

$$\theta' = \frac{q'}{h_{rc}A_t}, \quad (22)$$

where h_{rc} = combined radiation and convection thermal conductance.

[9] *Ibid.*, ch. 5, Table 2.

The part of (21) which is a function of z represents sinusoidal heating and cooling of the waveguide, relative to the heating, caused by q'. The temperature rises above ambient, represented by each portion of (21), θ' caused by q' and θ'' caused by $q'' \cos(\psi - 2\beta z)$, can be added, since the superposition theorem is assumed to be valid. That is, the temperature rise

$$\Delta T_p = \theta' + \theta''. \quad (23)$$

To find θ'', the following analysis is presented. Fig. 11 illustrates the summation of heat in a section Δz of the waveguide. The heat generated in the elemental section Δz of the guide, plus the heat flowing along the guide into the section, plus the heat from the surrounding air, is equal to zero[10] or

$$kA_c\frac{d^2\theta''}{dz^2} - h_{rc}A_t\theta'' - 2q_i|\Gamma| \cos\xi = 0, \quad (24)$$

where

$\theta'' = \theta''(z) = T_w(z) - T_a$ = temperature rise above ambient temperature,
k = thermal conductivity of waveguide wall,
A_c = cross-sectional area of waveguide,
A_t = total outer surface area of a unit length of waveguide, and
$\xi = \psi - 2\beta z$.

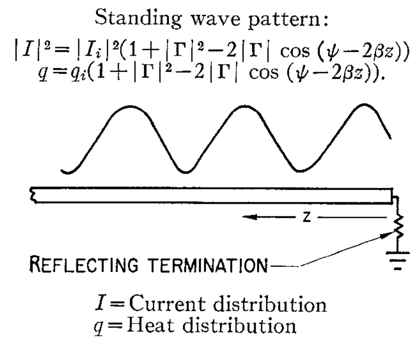

Fig. 10—Standing-wave pattern on a mismatched transmission line.

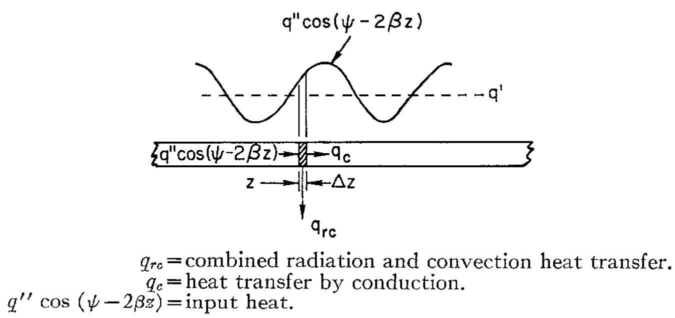

Fig. 11—Sinusoidal heat transfer relations of a mismatched transmission line.

[10] Refer to M. Jakob and G. A. Hawkins, "Elements of Heat Transfer," John Wiley and Sons, Inc., New York, N. Y., p. 157; 1957.

If the variable[11] of (24) is changed into

$$\frac{d^2\theta''}{dz^2} = \left(\frac{d\xi}{dz}\right)^2 \frac{d^2\theta''}{d\xi^2} = \left(\frac{4\pi}{\lambda}\right)^2 \frac{d^2\theta''}{d\xi^2}, \tag{25}$$

the temperature distribution θ'' along the waveguide can be determined by

$$kA_c\left(\frac{4\pi}{\lambda}\right)^2 \frac{d^2\theta''}{d\xi^2} - h_{rc}A_t\theta'' - 2q_i\left|\Gamma\right| \cos\xi = 0. \tag{26}$$

Eq. (26) is a nonhomogenous linear equation with the complementary function, or the transient term, equal to zero. The complementary function is zero, since the waveguide is assumed to be immersed in a uniform medium and is infinitely long.

A particular solution of (26) may be found[12] by assuming a particular integral and verifying by substitution. Thus,

$$\theta'' = \frac{-2q_i\left|\Gamma\right| \cos(\psi - 2\beta z)}{kA_c\left(\frac{4\pi}{\lambda}\right)^2 + h_{rc}A_t}. \tag{27}$$

Case 1—Equal Power Delivered to Either Matched or Mismatched Loads: A matched or a mismatched waveguide, having the same power W delivered to the load, has the following relations as derived from the transmission line equations:[1]

$$W = I^2_{\text{match}}Z_0 = I_{\max}I_{\min}Z_0 \tag{28}$$

$$\frac{I_{\max}}{I_{\min}} = \frac{I\sqrt{\rho}}{\frac{I}{\sqrt{\rho}}} = \rho = \text{VSWR}, \tag{29}$$

$$I_i = \frac{I}{2}\left[\sqrt{\rho} + \frac{1}{\sqrt{\rho}}\right]. \tag{30}$$

The "incident heat" q_i is proportional to $\left|I_i\right|^2$. If q_i is substituted into (22) and (27) with the reflection coefficient expressed in terms of VSWR, (8) expresses the waveguide maximum temperature rise corresponding to the high-current point on the mismatched waveguide with equal power dissipated in the load.

Case 2—Equal Incident Wave for Either Matched or Mismatched Loads: A mismatched waveguide having the same incident current as a matched load on the waveguide, or $I_{\text{match}} = I_i$ for various ρ, will have a derating factor for temperature rise different from Case 1. If (22) and (27) are expressed as a function of VSWR instead of the reflection coefficient, (11) expresses the waveguide temperature rise at the high temperature point resulting from a mismatched termination; however, (11) is valid, in this instance, only when the incident signal I_i for various $\rho = I_{\text{match}}$.

[11] W. W. Macalpine, *op. cit.*, (25A).

[12] For example, M. Morris and O. E. Brown, "Differential Equations," Prentice-Hall, Inc., New York, N. Y., p. 91; 1942.

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